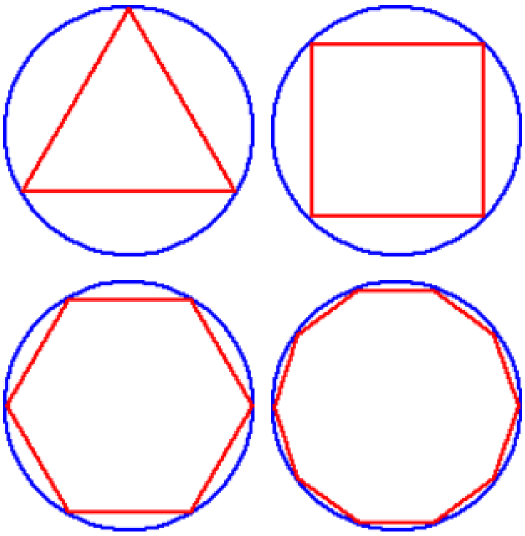
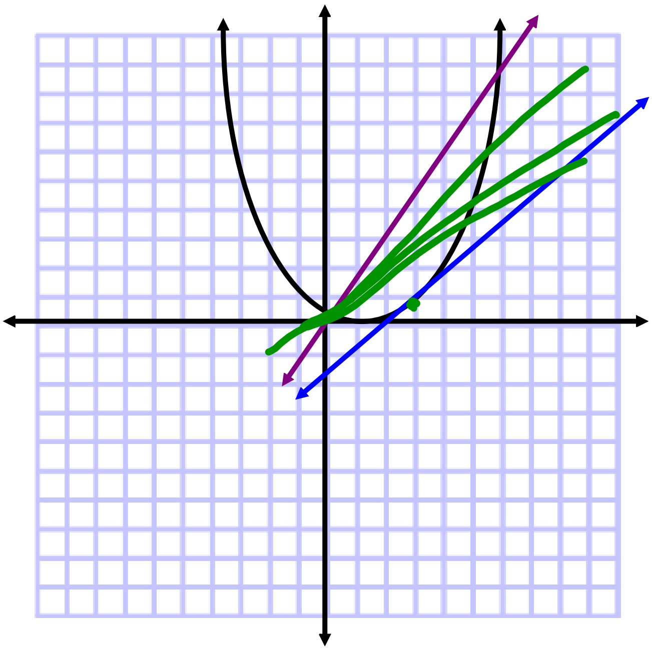
Limits





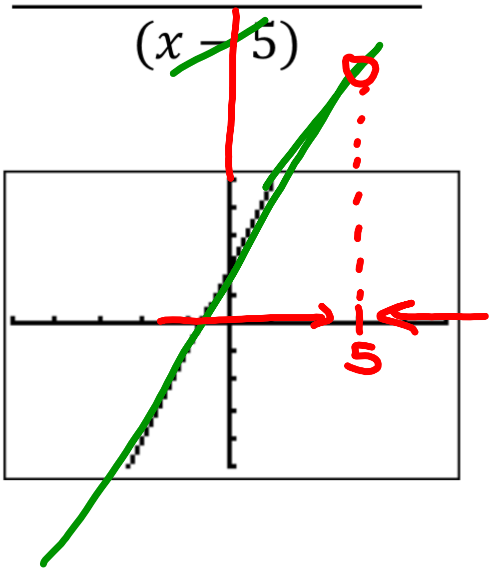
Consider the graph:

$(3x+2)\cancel{(x-5)}$

$$f(x) = \frac{(3x^2 - 13x - 10)}{\cancel{(x-5)}}$$

hole (5, 17)

(undefined)

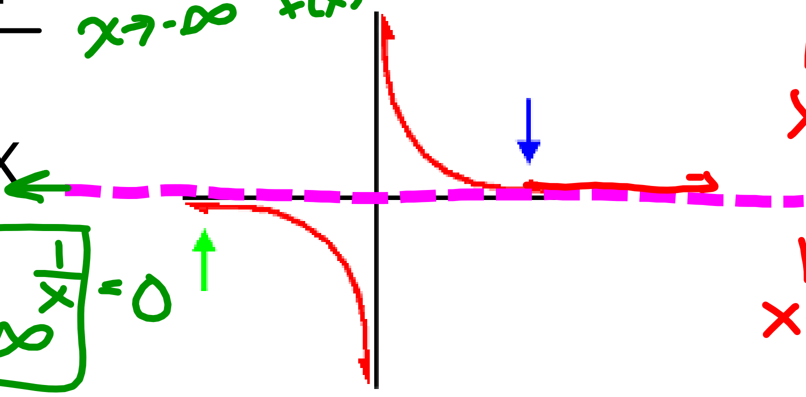


$\lim_{x \to 5} f(x) = 17$

$\frac{1}{X}$

end beh.

$x \to -\infty \quad f(x) \to 0$



$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to \infty} \frac{1}{x} = 0$

$\boxed{\lim_{x \to -\infty} \frac{1}{x}} = 0$

Limit Notation & Description

Suppose that *f* is a function defined on some open interval containing the number *a*. The function *f* may or may not be defined at a. The limit notation

$$\lim_{x \to a} f(x) = L$$

is read "the limit of $f(x)$ as *x* approaches *a* equals the number *L*." This means that as *x* gets closer to *a*, but remains unequal to *a*, the corresponding values of *f(x)* get closer to *L*.

Table:

$(x+1)\cancel{(x-1)}$

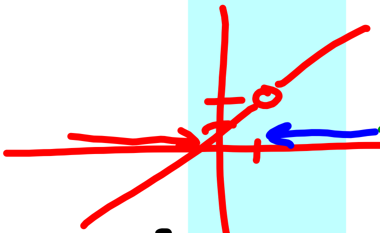
$$\frac{x^2-1}{x-1}$$

hole = 1

(1, 2)

$\lim\limits_{x\to 1^-} \frac{x^2-1}{x-1} = 2$

x		$\frac{(x^2-1)}{(x-1)}$
0.5		1.50000
0.9		1.90000
0.99		1.99000
0.999		1.99900
0.9999		1.99990
0.99999		1.99999
...		...



$\lim\limits_{x\to 1^+} \frac{x^2-1}{x-1} = 2$

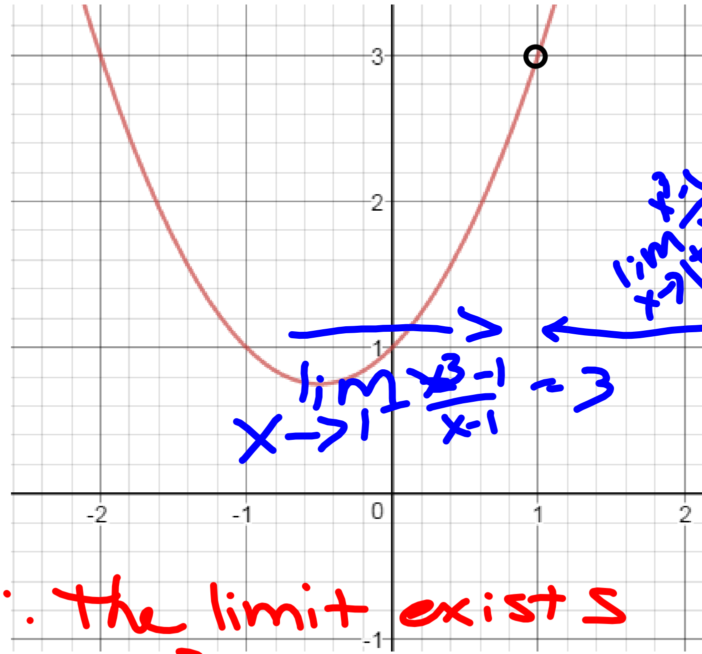
$\lim\limits_{x\to 1^-} \frac{x^2-1}{x-1} = 2$

When a limit approaches from the left and the right, they must equal the same # for the limit to exist.

$$\frac{x^3-1}{x-1}$$

$$\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}}$$

hole $(1,3)$

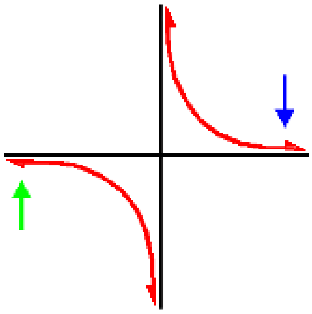


$$\lim_{x \to 1^-} \frac{x^3-1}{x-1} = 3$$

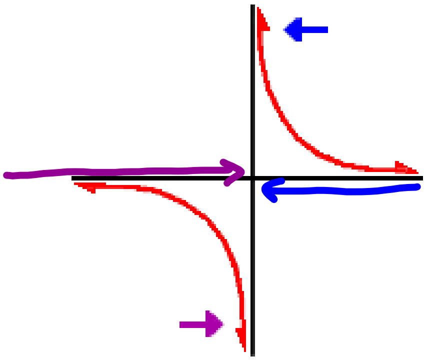
$$\lim_{x \to 1^+} \frac{x^3-1}{x-1} = 3$$

$\therefore$ the limit exists

$$\lim_{x \to 1} \frac{x^3-1}{x-1} = 3$$



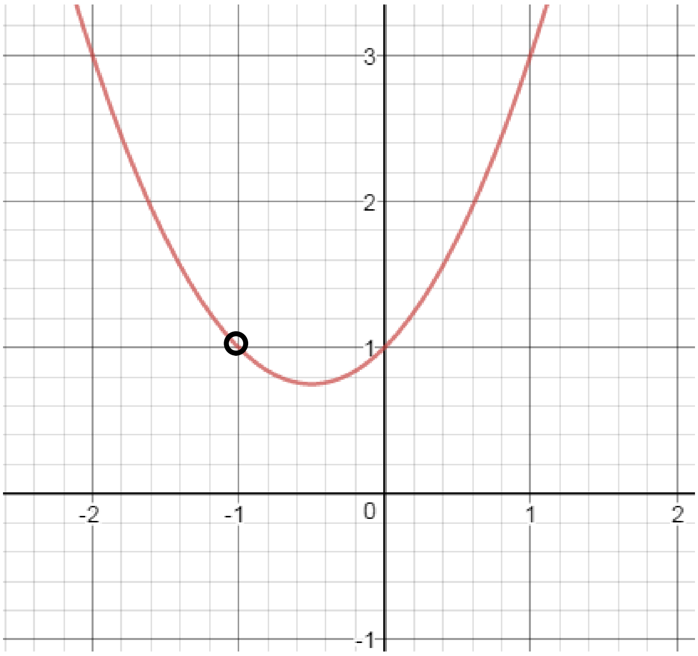
$$\lim_{x \to +\infty}\left(\frac{1}{x}\right) = 0 \qquad \lim_{x \to -\infty}\left(\frac{1}{x}\right) = 0$$

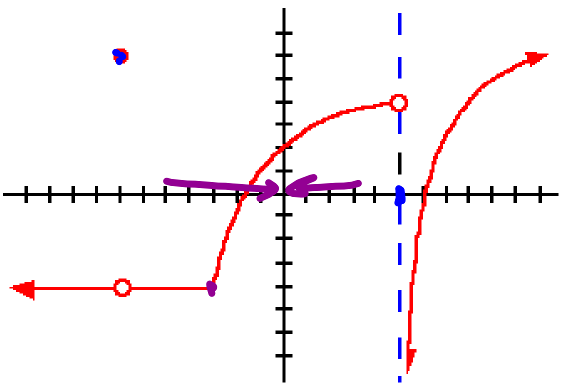


$$\lim_{x \to 0^-} \left(\frac{1}{x}\right) = -\infty \quad \text{and} \quad \lim_{x \to 0^+} \left(\frac{1}{x}\right) = \infty$$

$$\lim_{x \to 0} \left(\frac{1}{x}\right) = \begin{matrix}\text{does} \\ \text{not} \\ \text{exist}\end{matrix}$$

$$\frac{x^3 - 1}{x - 1}$$





$f(-7) = 6$

1. $\lim_{x \to 5^+} f(x) = -\infty$

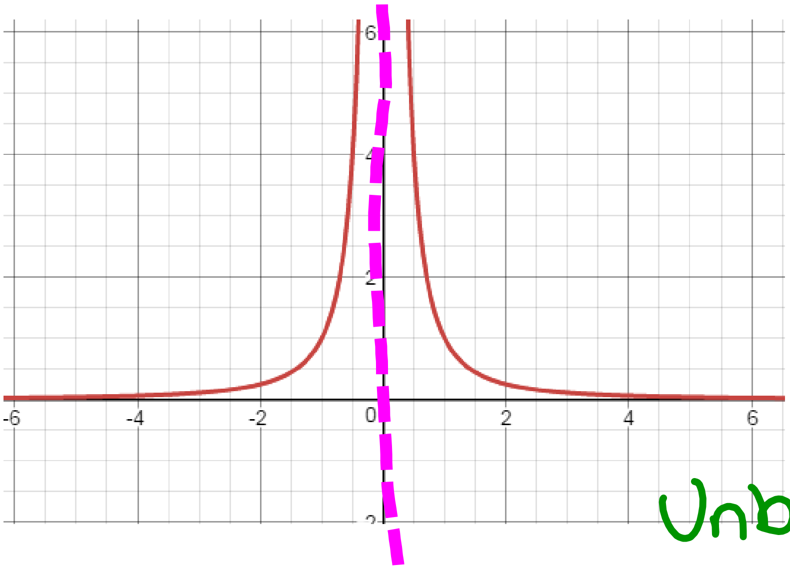
   $\lim_{x \to 5^-} f(x) = 4$

   $\lim_{x \to 5} f(x) =$ DNE

2. $\lim_{x \to -7} f(x) = -4$

3. $\lim_{x \to -3} f(x) = -4$

4. $\lim_{x \to 0} f(x) = 2$



$\lim_{x\to 0^-} \to \infty$

$\lim_{x\to 0^+} = \infty$

Unbound limit

DNE

Table:

$$\frac{x^2-1}{x-1}$$

x		$\frac{(x^2-1)}{(x-1)}$
0.5		1.50000
0.9		1.90000
0.99		1.99000
0.999		1.99900
0.9999		1.99990
0.99999		1.99999
...		...

x		$\frac{(x^2-1)}{(x-1)}$
1.5		2.50000
1.1		2.10000
1.01		2.01000
1.001		2.00100
1.0001		2.00010
1.00001		2.00001
...		...

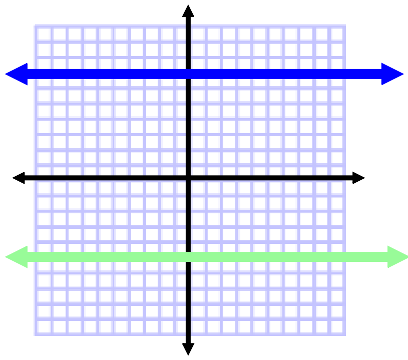
Limit of a Constant

The limit of a constant is that constant. For the constant function $f(x) = c, \lim_{x \to a} c = c$

Examples:

$\lim_{x \to 4} 7 = 7$

$\lim_{x \to 0} (-5) = -5$



Limit of a Polynomial

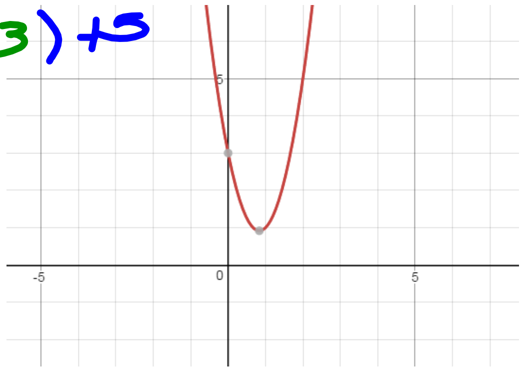
$$\lim_{x\to a} f(x) = f(a)$$

Examples:

$$\lim_{x\to 3}(4x^3 + 2x^2 - 6x + 5) = 113$$

$4(3)^3 + 2(3)^2 - 6(3) + 5$

$$\lim_{x\to 2}(3x^2 - 5x + 3)$$



Finding Limits When the Limit of the Denominator is Zero

Using Factoring:

$$\lim_{x\to 3}\frac{x^2-x-6}{x-3} \quad \frac{\cancel{(x-3)}(x+2)}{\cancel{x-3}} = 5$$

① factor & simplify first

$$\lim_{x\to 1}\frac{x^2-2x-3}{x-1}$$

More with Factoring

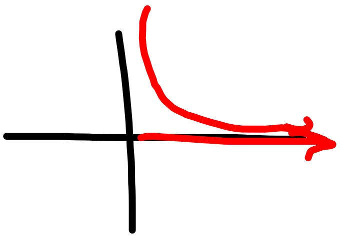
$$\lim_{x \to 5} \frac{x-5}{x^2-3x-10} = \frac{1}{7} \qquad \frac{\cancel{x-5}}{\cancel{(x-5)}(x+2)} \qquad \frac{1}{(5+2)} \qquad \frac{1}{7}$$

$$\lim_{x \to -5} \frac{x^2+x-20}{x^2+2x-15} \qquad \frac{\cancel{(x+5)}(x-4)}{\cancel{(x+5)}(x-3)} \qquad \frac{(-5-4)}{(-5-3)}$$

$$\lim\ldots = \frac{9}{8}$$

Going to Infinity

Consider horizontal asymptotes.

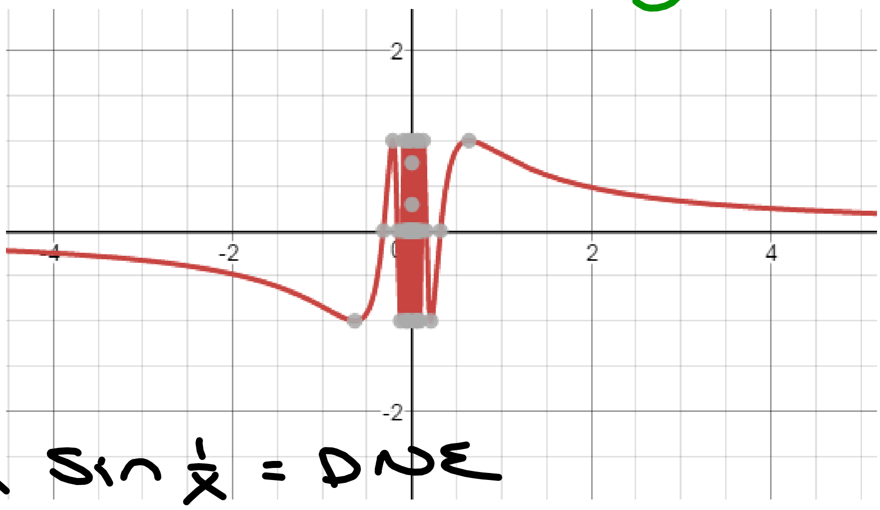


$$\lim_{x\to\infty} \frac{1}{x} = 0$$ HA $y=0$

$$\lim_{x\to\infty} \frac{2x^2-3}{x^2+2} = 2$$

sin 1/x

Oscilating graph

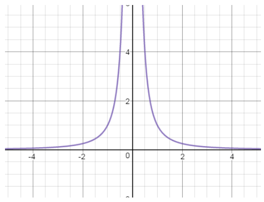


$$\lim_{x \to 0} \sin \frac{1}{x} = DNE$$

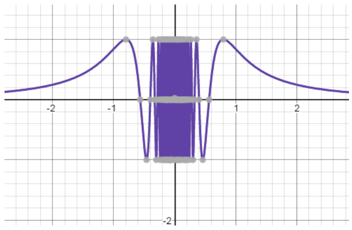
Limits Don't exist when:

1. The limit from the left and right are not equal

2. Unbounded behavior



3. Oscillating behavior



$$f(x) = \begin{cases} -1, & x < -1 \\ -\frac{x}{2} - \frac{1}{2}, & x \geq -1 \end{cases}$$

$$\lim_{x \to -1} f(x) = DNE$$